MENIIT
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JEE MAINS-2015

10-04-2015 (Online-1)

IMPORTANT INSTRUCTIONS

1. The test is of **3** hours duration.
2. The Test Booklet consists of **90** questions. The maximum marks are **360**.
3. There are **three** parts in the question paper A, B, C consisting of **Physics, Chemistry and Mathematics** having 30 questions in each part of equal weightage. Each question is allotted **4 (four)** marks for each correct response.
4. Candidates will be awarded marks as stated above in instruction No.3 for correct response of each question. 1/4 (one fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
5. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 4 above.

PART-A-PHYSICS

1. A telescope has an objective lens of focal length 150 cm and an eyepiece of focal length 5 cm. If a 50 m tall tower at a distance of 1 km is observed through this telescope in normal setting, the angle formed by the image of the tower is θ, then θ is close to

(1) 1° (2) 15° (3) 30° (4*) 60°

Sol. $MP = \frac{f_0}{f_e} = \frac{150}{5} = 30$

$\theta_0 = \frac{50}{1000} = \frac{1}{20} rad$

θ_0 = MP × θ_0

$= 30\frac{1}{20}$

$= \frac{3}{2} = 1.5 rad$

$= 1.5 \times \frac{180}{\pi} = 86°$

2. A very long (length L) cylindrical galaxy is made of uniformly distributed mass and has radius R (R < < L). A star outside the galaxy is orbiting the galaxy in a plane perpendicular to the galaxy and passing through its centre. If the time period of star is T and its distance from the galaxy's axis is r, then :

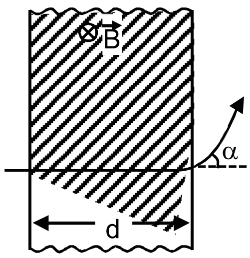
(1) $T \propto r^2$ (2*) $T \propto r$ (3) $T \propto \sqrt{r}$ (4) $T^2 \propto r^3$

Sol. $F = \frac{2GM}{Lr}m$

$mr\left(\frac{2\pi}{T}\right)^2 = \frac{2GMm}{Lr}$

$T \propto r$

3. A proton (mass m) accelerated by a potential difference V flies through a uniform transverse magnetic field B. The field occupies a region of space by width 'd'. If 'α' be the angle of deviation of proton from initial direction of motion (see figure), the value of sin α will be :



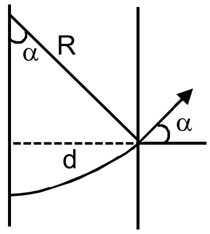
(1*) $Bd\sqrt{\frac{q}{2mV}}$ (2) $qV\sqrt{\frac{Bd}{2m}}$ (3) $\frac{B}{2}\sqrt{\frac{qd}{mV}}$ (4) $\frac{B}{d}\sqrt{\frac{q}{2mV}}$

Sol. $\sin\alpha = \frac{d}{R}$

$\sin\alpha = \frac{dqB}{mu}$

$\sin\alpha = Bd\sqrt{\frac{q}{2mV}}$

$\therefore \left[qV = \frac{1}{2}mu^2\right]$



4. A bat moving at 10 ms^{-1} towards a wall sends a sound signal of 8000 Hz towards it. On reflection it hears a sound of frequency f. The value of f in Hz is close to (speed of sound = 320 ms^{-1})

(1) 8258 (2) 8424 (3*) 8516 (4) 8000

Sol. Frequency incident on wall $= \frac{V}{V-10}f$

Reflected frequency reaching bat $= \frac{V+10}{V-10}f = 8516\text{Hz}$

5. An electromagnetic wave travelling in the x-direction has frequency of 2 × 10^{14} Hz and electric field amplitude of 27 Vm^{-1}. From the options given below, which one describes the magnetic field for this wave?

(1) $\vec{B}(x, t) = (9 \times 10^{-8} T)\ \hat{j}$

$\sin[1.5 \times 10^{-6} x - 2 \times 10^{14} t]$

(2) $\vec{B}(x, t) = (3 \times 10^{-8} T)\ \hat{j}$

$\sin[2\pi (1.5 \times 10^{-8} x - 2 \times 10^{14} t]$

(3) $\vec{B}(x, t) = (9 \times 10^{-8} T)\ \hat{i}$

$\sin[2\pi (1.5 \times 10^{-8} x - 2 \times 10^{14} t]$

(4*) $\vec{B}(x, t) = (9 \times 10^{-8} T)\hat{k}$

$\sin[2\pi (1.5 \times 10^{-6} x - 2 \times 10^{14} t]$

Sol. $\omega = 2\pi \times 2 \times 10^{14}$ Hz

$B_0 = \frac{E_0}{C} = \frac{27}{3\times10^8} = 9\times10^{-8}$ Tesla

Oscillation of B can be only along $\hat{j}$ or $\hat{k}$ direction.

$\therefore$ Option (3)

6. When current in a coil changes from 5A to 2 A in 0.1 s, an average voltage of 50 V is produced. The self - inductance of the coil is :

(1*) 1.67 H (2) 0.67 H (3) 6 H (4) 3 H

Sol. $\text{EMF} = \frac{Ldi}{dt}$

$$50 = L\left(\frac{5-2}{0.1sec}\right)$$

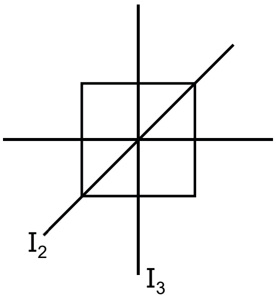
$$\Rightarrow L = \frac{50 \times 0.1}{3} = \frac{5}{3} = 1.67H$$

7. Consider a thin uniform square sheet made of a rigid material. If its side is 'a', mass m and moment of inertia I about one of its diagonals, then :

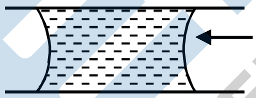
(1) $I = \frac{ma^2}{24}$ (2) $\frac{ma^2}{24} < I < \frac{ma^2}{12}$ (3) $I > \frac{ma^2}{12}$ (4*) $I = \frac{ma^2}{12}$

Sol. For uniform thin square sheet

$$I_1 = I_2 = I_3 = \frac{ma^2}{12}$$



8. If two glass plates have water between them and are separated by very small distance (see figure), it is very difficult to pull them apart. It is because the water in between forms cylindrical surface on the side that gives rise to lower pressure in the water in comparison to atmosphere. If the radius of the cylindrical surface is R and surface tension of water is T then the pressure in water between the plates is lower by:



(1*) $\frac{T}{R}$ (2) $\frac{4T}{R}$ (3) $\frac{2T}{R}$ (4) $\frac{T}{4R}$

Sol. Excess pressure $= T\left[\frac{1}{r_1} + \frac{1}{r_2}\right]$

$$= \frac{T}{R} \because \left(\begin{matrix} r_1 = R \\ r_2 = 0 \end{matrix}\right)$$

9. If one were to apply Bohr model to a particle of mass 'm' and charge 'q' moving in a plane under the influence of a magnetic field 'B', the energy of the charged particle in the n^{th} level will be

(1) $n\left(\frac{hqB}{8\pi m}\right)$ (2) $n\left(\frac{hqB}{2\pi m}\right)$ (3) $n\left(\frac{hqB}{\pi m}\right)$ (4*) $n\left(\frac{hqB}{4\pi m}\right)$

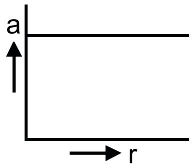
Sol. $qVB = \frac{mv^2}{r}$ ...(i)

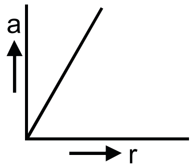
$\frac{nh}{2\pi} = mvr$ ....(ii)

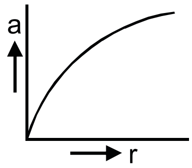
$\text{multiply both} \frac{qBnh}{2\pi} = m^2v^2$

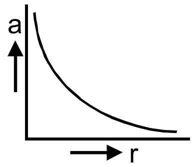
$KE = n\left[\frac{qBh}{4\pi m}\right]$

10. If a body moving in a circular path maintains constant speed of 10 ms^{-1}, then which of the following correctly describes relation between acceleration and radius?

(1) 

(2) 

(3) 

(4*) 

Sol. V = constant

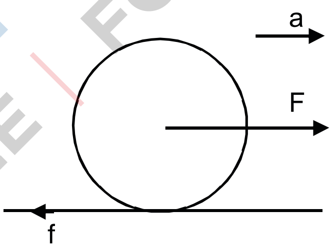
$a = \frac{V^2}{r}$

r_a = constant

11. A uniform solid cylindrical roller of mass 'm' is being pulled on a horizontal surface with force F parallel to the surface and applied at its centre. If the acceleration of the cylinder is 'a' and its is rolling without slipping then the value of 'F' is

(1) 2 ma (2) ma (3*) $\frac{3}{2}$ ma (4) $\frac{5}{3}$ ma

Sol. ma = F – f ....(i)



$\frac{mR^2}{2} \cdot \alpha = fR$

$\frac{mR^2}{2}\frac{a}{R} = fR$

$\frac{ma}{2} = f$ ....(ii)

Put in equation (i)

$ma = F - \frac{ma}{2}$

$F = \frac{3ma}{2}$

12. A simple harmonic oscillator of angular frequency 2 rad s^{-1} is acted upon by an external force F = sin t N. If the oscillator is at rest in its equilibrium position at t = 0, its position at later times is proportional to

(1) sint + $\frac{1}{2}$ cos 2t (2*) sin t – $\frac{1}{2}$ sin 2t (3) cos t – $\frac{1}{2}$ sin 2t (4) sin t + $\frac{1}{2}$ sin 2t

Sol. F = ms

a $\propto$ sin t

$\frac{dv}{dt} \propto \sin t$

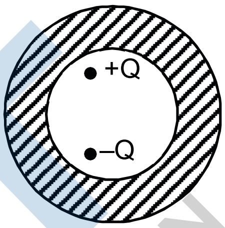
$\int_0^0 dV \propto \int_0^t \sin t \, dt$

$V \propto -\cos t + 1$

$\int_0^x dx = \int_0^t (-\cos y + 1)dt$

x =-sin t +1

13. Shown in the figure are two point charges + Q and – Q inside the cavity of a spherical shell. The charges are kept near the surface of the cavity on opposite sides of the centre of the shell. If σ_1 is the surface charge on the inner surface and Q_1 net charge on it and σ_2 the surface charge on the outer surface and Q_2 net charge on it then



(1) $\sigma_1 \neq 0, Q_1 = 0$

$\sigma_2 \neq 0, Q_2 = 0$

(2*) $\sigma_1 \neq 0, Q_1 \neq 0$

$\sigma_2 \neq 0, Q_2 \neq 0$

(3) $\sigma_1 = 0, Q_1 = 0$

$\sigma_2 = 0, Q_2 = 0$

(4*) $\sigma_1 \neq 0, Q_1 = 0$

$\sigma_2 = 0, Q_2 = 0$

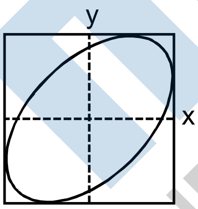
Sol. Net charge inside cavity is zero

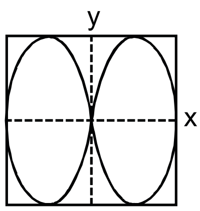
$\therefore Q_1 = 0$ and $\sigma_1 = 0$

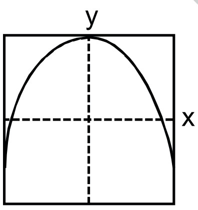
There is no effect of +Q, –Q and induced charge on inner surface on the outer surface hence

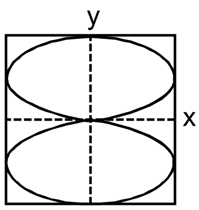
$Q_2 = 0, \sigma_2 = 0$

14. x and y displacements of a particle are given as x(t) = a sin ωt and y(t) = a sin 2ωt. Its trajectory will look like :

(1) 

(2*) 

(3) 

(4*) 

Sol. At t = 0, x(t) = 0 ; y(t) = 0

x(t) is a

sinusoidal function hence option (1)

At $t = \frac{\pi}{2\omega}$

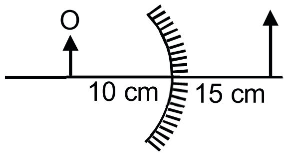
x (t) = a

y (t) = 0

15. You are asked to design a shaving mirror assuming that a person keeps it 10 cm from his face and views the magnified image of the face at the closest comfortable distance of 25 cm. The radius of curvature of the mirror would then be :

(1) 30 cm (2*) 60 cm (3) 24 cm (4) – 24 cm

Sol. $\frac{1}{15} + \frac{1}{(-10)} = \frac{1}{f}$



F = -30 cm

R = -60 cm

16. A block of mass m = 0.1 kg is connected to a spring of unknown spring constant k. It is compressed to a distance x from its equilibrium position and released from rest. After approaching half the distance $\left(\frac{x}{2}\right)$ from equilibrium position, it hits another block and comes to rest momentarily, while the other block moves with a velocity 3 ms^{-1}. The total initial energy of the spring is

(1) 0.8 J (2) 1.5 J (3*) 0.6 J (4) 0.3 J

Sol. Apply momentum conservation

0.1u + m(0) = 0.1(0) + m(3)

$\frac{1}{2}0.4u^2 = \frac{1}{2}m(3)^2$

Solving u = 3

$\frac{1}{2}kx^2 = \frac{1}{2}K\left(\frac{x}{2}\right)^2 6\frac{1}{2}(0.1)3^2$

$\frac{3}{4}Kx^2 = 0.9$

$\frac{1}{2}Kx^2 = 0.6J$

17. A 25 cm long solenoid has radius 2 cm and 500 total number of turns. It carries a current of 15 A. If it is equivalent to a magnet of the same size and magnetization $\vec{M}$ (magnetic moment/volume), then || $\vec{M}$ is :

(1) 30000 π Am^{-1} (2*) 30000 Am^{-1} (3) 300 Am^{-1} (4) 3π Am^{-1}

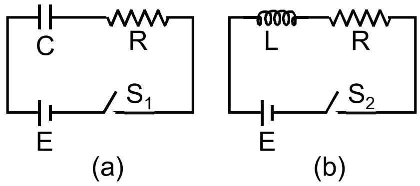
Sol. $\vec{m}(\text{mag.moment / volume}) = \frac{NiA}{A\ell}$

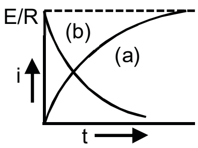
$= \frac{Ni}{\ell}$

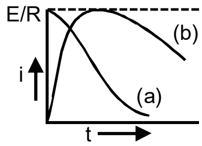
$= \frac{(500)15}{25 \times 10^{-2}}$

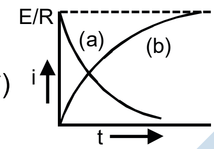
= 30000 Am^{-1}

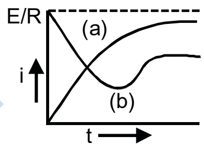
18. In the circuits (a) and (b) switches S_1 and S_2 are closed at t = 0 and are kept closed for a long time. The variation of currents in the two circuits for $t \geq 0$ are roughly shown by (figures are schematic and not drawn to scale :



(1) 

(2) 

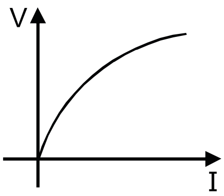
(3*) 

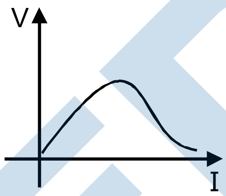
(4) 

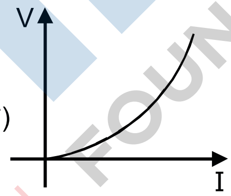
Sol. For capacitor circuit $i = i_0 e^{-t/RC}$

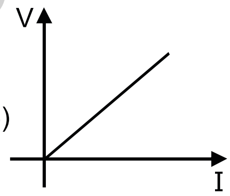
For capacitor circuit $i = i_0\left(1 - e^{\frac{Rt}{L}}\right)$

19. Suppose the drift velocity v_d in a material varied with the applied electric field E as $v_d \propto \sqrt{E}$. Then V – I graph for a wire made of such a material is best given by :

(1) 

(2) 

(3*) 

(4) 

Sol. $i = neAV_d$

$i \propto \sqrt{E}$

$i^2 \propto E$

$i^2 \propto V$

20. If the capacitance of a nanocapacitor is measured in terms of a unit 'u' made by combining the electronic charge 'e', Bohr radius 'a_0', Planck's constant 'h' and speed of light 'c' then :

(1) $u = \frac{e^2c}{ha_0}$ (2) $u = \frac{e^2h}{ca_0}$ (3*) $u = \frac{e^2a_0}{hc}$ (4) $u = \frac{hc}{e^2a_0}$

Sol. $[u] = [e]^a[a_0]^b[h]^c[c]^d$

$[M^{-1}L^{-2}T^{+4}A^{+2}] = [A^1T^1]^a\ [L]^b[ML2T^{-1}]^c[LT^{-1}]^d$

$[M^{-1}L^{-2}T^{+4}A^{+2}] = [M^C L^{b+2c+d}\ T^{a-c-d} A^a]$

a = 2, b = 1, c = – 1, d = – 1

$u = \frac{e^2a_0}{hc}$

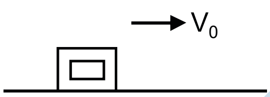
21. A block of mass m = 10 kg rests on a horizontal table. The coefficient of friction between the block and the table is 0.05. When hit by a bullet of mass 50 g moving with speed v, that gets embedded in it, the block moves and comes to stop after moving a distance of 2 m on the table. If a freely falling object were to acquire speed $\frac{v}{10}$ after being dropped from height H, then neglecting energy losses and taking $g = 10\ ms^{-2}$, the value of H is close to :

(1) 0.5 km (2) 0.3 km (3) 0.2 km (4) 0.4 km

Ans. (Bonus)

Sol. Momentum conservation $0.05V = 10V_0$

$0 - V_0^2 = 2(-\mu g)2$



$V_0 = \sqrt{2}$

object falling from height H.

$\frac{V}{10} = \sqrt{2gH}$

H = 40 m = 0.04 km

No option match (Bonus)

22. If it takes 5 minutes to fill a 15 litre bucket from a water tap of diameter $\frac{2}{\sqrt{\pi}}$ cm then the Reynolds number for the flow is (density of water = 10^3 kg/m^3 and viscosity of water = 10^{-3} Pa.s) close to :

(1) 550 (2*) 5500 (3) 11,000 (4) 1100

Sol. $\frac{dm}{dt} = SAV$

$\frac{15}{5 \times 60} = 10^3 \times \pi\left(\frac{1}{\sqrt{\pi}}\right)^2 \times 10^{-4} V$

V = 0.05 m/s

$R_e = \frac{SVD}{\mu}$

$= \frac{10^3 \times 0.5 \times \frac{2}{\sqrt{\pi}} 10^{-2}}{10^{-3}}$

= 5500 **Ans.**

23. In an ideal gas at temperature T, the average force that a molecule applies on the walls of a closed container depends on T as T^q. A good estimate for q is :

(1) 2 (2) $\frac{1}{4}$ (3) $\frac{1}{2}$ (4*) 1

Sol. $\text{Pressuere}\ \alpha V_{rms}^2$

$\text{Force}\ \alpha V_{rms}^2 \propto T$

24. Diameter of a steel ball is measured using a Vernier callipers which has divisions of 0.1 cm on its main scale (MS) and 10 divisions of its vernier scale (VS) match 9 divisions on the main scale. Three such measurements for a ball are given as :

S.No.	MS(cm)	VS divisions
1.	0.5	8
2.	0.5	4
3.	0.5	6

If the zero error is – 0.03 cm, then mean corrected diameter is :

(1) 0.56 cm (2) 0.53 cm (3*) 0.59 cm (4) 0.52 cm

Sol. Least count = 0.01 cm

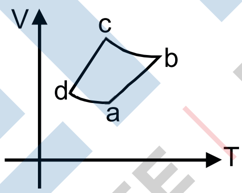
$d_1 = 0.5 + 8 \times 0.01 + 0.03 = 0.61$ cm

$d_2 = 0.5 + 4 \times 0.01 + 0.03 = 0.57$ cm

$d_3 = 0.5 + 6 \times 0.01 + 0.03 = 0.59$ cm

$$\text{Mean diameter} = \frac{0.61 + 0.57 + 0.59}{2} = 0.59\,\text{cm}$$

25. An ideal gas goes through a reversible cycle a → b → c → d has the V - T diagram shown below. Process d → a and b → c are adiabatic The corresponding P - V diagram for the process is (all figures are schematic and not drawn to scale)



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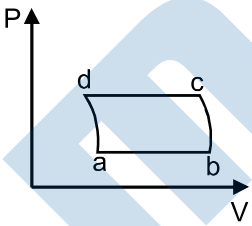

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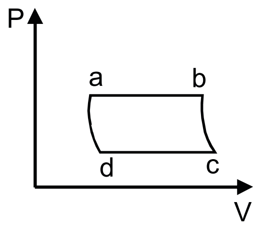

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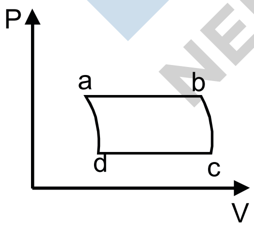

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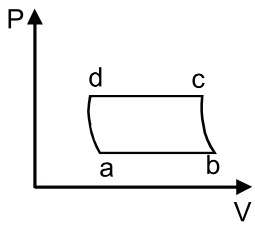


Sol. In VT graph

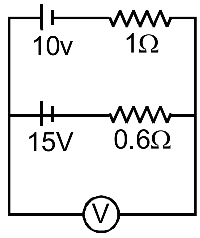
ab-process : Isobaric line passes through origin, temperature increases.

bc process : Adiabatic, pressure decreases.

cd process : isobaric, volume decreases.

da process : Adiabatic, pressure increase.

26. A 10 V battery with internal resistance 1 Ω and a 15 V battery with internal resistance 0.6Ω are connected in parallel to a voltmeter (see figure). The reading in the voltmeter will be close to :



(1) 13.1 V (2*) 12.5 V (3) 24.5 V (4) 11.9 V

Sol. $E = \dfrac{\frac{10}{1}+\frac{15}{06}}{\frac{1}{1}+\frac{1}{0.6}} = \dfrac{10+25}{2.67} = 13.1V$

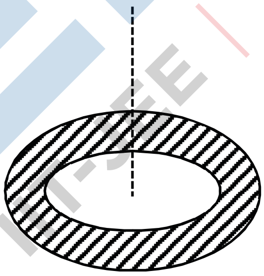
27. de-Broglie wavelength of an electron accelerated by a voltage of 50 V is close to ($|e| = 1.6 \times 10^{-19}$ C, $m_e = 9.1 \times 10^{-31}$ kg, $h = 6.6 \times 10^{-34}$ Js)

(1) 1.2 Å (2) 2.4 Å (3) 0.5 Å (4*) 1.7 Å

Sol. $\lambda = \dfrac{h}{mv} = \dfrac{h}{\sqrt{2mqV}}$

= 1.7 Å

28. A thin disc of radius b = 2a has concentric hole of radius 'a' in it (see figure). It carries uniform surface charge 'σ' on it. If the electric field on its axis at height 'h' (h << a) from its centre is given as 'Ch' then value of 'C' is :



(1) $\dfrac{\sigma}{a\varepsilon_0}$ (2) $\dfrac{\sigma}{2a\varepsilon_0}$ (3) $\dfrac{\sigma}{4a\varepsilon_0}$ (4) $\dfrac{\sigma}{8a\varepsilon_0}$

Sol. Electric field due to complete disc (R = 2a)

$$E_1 = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{R^2+x^2}}\right]$$

$$E_1 = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{h}{\sqrt{4a^2+h^2}}\right] = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{h}{2a}\right]$$

Electric field due to disc (R = a)

$$E_2 = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{h}{a}\right)$$

Electric field due to given disc.

$E = E_1 - E_2$

$= \frac{\sigma h}{4\varepsilon_0 a}$

option (2)

29. In an unbiased n-p junction electrons diffuse from n - region to p - region because :

(1) holes in p - region attract them

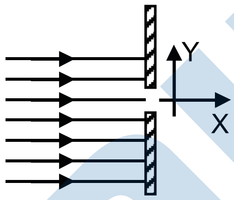
(2*) electron concentration in n - region is more as compared to that in p - region

(3) electrons travel across the junction due to potential difference

(4) only electrons move from n to p region and not the vice - versa

Sol. T

30. A parallel beam of electrons travelling in x-direction falls on a slit of width d (see figure). If after passing the slit, an electron acquires momentum p_y in the y-direction then for a majority of electrons passing through the slit (h is Planck's constant)



(1) $|p_y|\, d > h$ (2) $|p_y|\, d \approx h$ (3) $|p_y|\, d >> h$ (4) $|p_y|\, d < h$

Sol. $d \sin\theta = \lambda$.

$\sin\theta = \frac{\lambda}{d} < 1$

$\frac{h}{|P_y|} < d$

$h < |P_y|\, d$

Option (1)

PART-B-CHEMISTRY

31. Match the polymers in column-A with their main uses in column–B and choose the correct answer :

	Column – A		**Column–B**
(A)	Polystyrene	(i)	Paints and lacquers
(B)	Glyptal	(ii)	Rain coats
(C)	Polyvinyl Chloride	(iii)	Manufacture of toys
(D)	Bakelite	(iv)	Computer discs

(1) (A)–(ii), (B)–(i), (C)–(iii), (D)–(iv)
(2*) (A)–(iii), (B)–(iv), (C)–(ii), (D)–(i)
(3) (A)–(ii), (B)–(iv), (C)–(iii), (D)–(i)
(4) (A)–(iii), (B)–(i), (C)–(ii), (D)–(iv)

Sol. (A) - Manufacture of toys (iii)

(B) - Points & lacquers (i)

(C) - Rain coats (ii)

(D) - Complete discs. (iv)

32. Permanent hardness in water cannot be cured by :

(1) Calgon's method
(2) Ion exchange method
(3) Treatment with washing soda
(4*) Boiling

Sol. Boiling can remove only temporary hardness caused by bicarbonates of Ca^{2+}, Mg^{2+}.

33. An aqueous solution of a salt X turns blood red on treatment with SCN^- and blue on treatment with $K_4[Fe(CN)_6]$. X also gives a positive chromyl chloride test. The salt X is :

(1) $Cu(NO_3)_2$
(2*) $FeCl_3$
(3) $Fe(NO_3)_3$
(4) $CuCl_2$

Sol. $FeCl_3$ gives chromyl chloride test,

$Fe^{3+} + SCN^- \longrightarrow$ blood red color.

and Fe^{3+} $K_4[Fe(CN)_6]$ $Fe_4[Fe(CN)_6]_3$
(blue)

34. After understanding the assertion and reason, choose the correct option.

Assertion : In the bonding molecular orbital (MO) of H_2, electron density is increased between the nuclei.

Reason : The bonding MO is $\Psi_A + \Psi_B$, which shows destructive interference of the combining electron waves.

(1*) Assertion is correct, reason is incorrect.
(2) Assertion and reason are correct, but reason is not the correct explanation for the assertion.
(3) Assertion and reason are correct and reason is the correct explanation for the assertion.
(4) Assertion is incorrect, reason is correct.

Sol. Bonding molecular orbital results in increased electron density between nuclei due to constructive interference of combining electron waves.

35. Gaseous N_2O_4 dissociates into gaseous NO_2 according to the reaction $N_2O_4(g) \rightleftharpoons 2NO_2(g)$

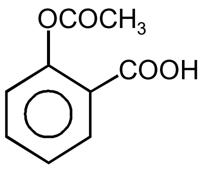
At 300 K and 1 atm pressure, the degree of dissociation of N_2O_4 is 0.2. If one mole of N_2O_4 gas is contained in a vessel, then the density of the equilibrium mixture is :

(1*) 3.11 g/L (2) 4.56 g/L (3) 6.22 g/L (4) 1.56 g/L

Sol. $\frac{M_{Th}}{M_{Ob}} = 1 + (2-1)\alpha = 1.2.$

$\Rightarrow \quad M_{Ob} = \frac{92}{1.2}$

and $\quad d = \frac{PM}{RT} = \frac{1 \times 92}{1.2 \times 0.082 \times 300} = 3.116\,g/L$

36.  is used as :

(1*) Analgesic (2) Insecticide (3) Antacid (4) Antihistamine

Sol. Aspirin is non-narcotic analgesic.

37. A sample of a hydrate of barium chloride weighing 61 g was heated until all the water of hydration is removed. The dried sample weighed 52 g. The formula of the hydrated salt is :

(Atomic mass, Ba = 137 amu, Cl = 35.5 amu)

(1) $BaCl_2.3H_2O$ (2) $BaCl_2.H_2O$ (3) $BaCl_2.4H_2O$ (4*) $BaCl_2.2H_2O$

Sol. $BaCl_2.xH_2O \longrightarrow BaCl_2 + xH_2O.$

$m_{H_2O} = 61 - 52 = 9g$

$n_{H_2O} = \frac{9}{18} = \frac{1}{2}$

$m_{BaCl_2} = 52 \Rightarrow \quad n_{BaCl_2} = \frac{52}{208} = \frac{1}{4} \Rightarrow \text{simplest formula} = \frac{1}{4} : \frac{1}{2} = 1:2 \Rightarrow \quad BaCl_2.2H_2O$

38. The cation that will not be precipitated by H_2S in the presence of dil HCl is :

(1) Pb^{2+} (2*) Co^{2+} (3) Cu^{2+} (4) As^{3+}

Sol. Co^{2+} is precipitated when we have sufficient S^{2-} concentration.

39. The least number of oxyacids are formed by :

(1) Sulphur (2*) Fluorine (3) Chlorine (4) Nitrogen

Sol. Fluorine only forms HOF as it cannot show multiple oxidation states.

40. The correct order of thermal stability of hydroxides is :

(1) $Ba(OH)_2 < Sr(OH)_2 < Ca(OH)_2 < Mg(OH)_2$

(2*) $Mg(OH)_2 < Sr(OH)_2 < Ca(OH)_2 < Ba(OH)_2$

(3) $Ba(OH)_2 < Ca(OH)_2 < Sr(OH)_2 < Mg(OH)_2$

(4) $Mg(OH)_2 < Ca(OH)_2 < Sr(OH)_2 < Ba(OH)_2$

Sol. Larger cation is able to stabilize polyatomic anion more than smaller cation.

41. Which molecule/ion among the following cannot act as a ligand in complex compounds?

(1) Br^- (2) CO (3) CN^- (4*) CH_4

Sol. CH_4 does not have lone pair.

42. 1.4 g of an organic compound was digested according to Kjeldahl's method and the ammonia evolved was absorbed in 60 mL of M/10 H_2SO_4 solution. The excess sulphuric acid required 20mL of M/10 NaOH solution for neutralization. The percentage of nitrogen in the compound is :

(1*) 10 (2) 24 (3) 5 (4) 3

Sol. Organic compound $\longrightarrow NH_3$

(1.4g)

$2H_2SO_4 + H_2SO_4 \longrightarrow (NH_4)2SO_4$

$H_2SO_4 + 2NaOH \longrightarrow Na_2SO_4 + 2H_2O$

$$n_{NH_3} + 20 \times \frac{1}{10} \times \frac{1}{1000} = 60 \times \frac{1}{10} \times 2 \times \frac{1}{1000}$$

$$n_{NH_3} = \frac{12}{1000} - \frac{2}{1000} = \frac{10}{100}$$

$$n_N = n_{NH_3} = 0.01 \Rightarrow m_N = 0.01 \times 14 = 0.14\,g \Rightarrow \%N \frac{0.14}{1.4} \times 10\%.$$

43. A variable, opposite external potential (E_{ext}) is applied to the cell

Zn | Zn^{2+} (1 M) || Cu^{2+} (1 M) | Cu, of potential 1.1 V. When E_{ext} < 1.1 V and E_{ext} > 1.1 V, respectively electrons flow from :

(1) cathode to anode in both cases

(2) anode to cathode in both cases

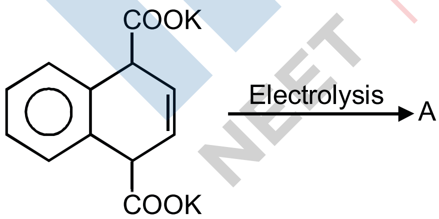
(3) cathode to anode and anode to cathode

(4*) anode to cathode and cathode to anode

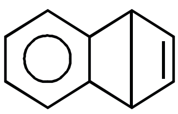
Sol. Electrons flow from anode to cathode always.

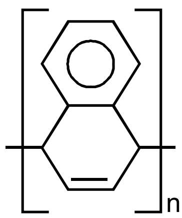
44. Photochemical smog consists of excessive amount of X, in addition to aldehydes, ketones, peroxy acetyl nitrile (PAN), and so forth. X is :

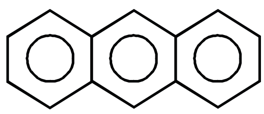
(1) CO (2*) CO_2 (3) O_3 (4) CH_4

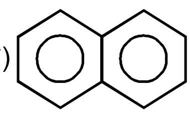
45. 

A is

(1) 

(2) 

(3) 

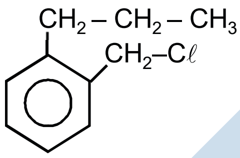
(4*) 

46. A compound A with molecular formula $C_{10}H_{13}Cl$ gives a white precipitate on adding silver nitrate solution. A on reacting with alcoholic KOH gives compound B as the main product. B on ozonolysis gives C and D. C gives Cannizaro reaction but not aldol condensation. D gives aldol condensation but not Cannizaro reaction. A is :

(1) $C_6H_5–CH_2–CH_2–CH_2–CH_2–Cl$

(2) $C_6H_5 – CH_2 – CH_2 – CH(Cl) – CH_3$

(3*) $C_6H_5 – CH_2 – C(Cl)(CH_3)(CH_3)$

(4) 

47. The heat of atomization of methane and ethane are 360 kJ/mol and 620 kJ/mol, respectively. The longest wavelength of light capable of breaking the C–C bond is

(Avogadro number = 6.02×10^{23}, h = 6.62×10^{-34} Js)

(1) 2.48×10^3 nm (2*) 1.49×10^3 nm (3) 2.48×10^4 nm (4) 1.49×10^4 nm

Sol. CH4(g) $\longrightarrow$ C(g) + 4H(g)

$\Rightarrow 4 \times E_{C\text{-}H} = 360$ KJ/Mol. $\Rightarrow E_{C\text{-}H} = 90$ KJ/Mol.

and $C_2H_6(g) \longrightarrow 2C(g) + 6H(g)$

$\Rightarrow E_{C\text{-}C} + 6 \times 90 = 620$ $\Rightarrow E_{C\text{-}C} = 80$ kJ/mol

$$\Rightarrow N_A \times \frac{hc}{\lambda} = 80 \times 1000\,J$$

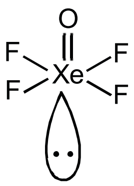
$$\lambda = \frac{60.02 \times 10^{23} \times 10^{-34} \times 3 \times 10^{8}}{80000}$$

$= 14.9 \times 10^{-7}$ m = 1.49×10^{-6} m

$= 1.49 \times 10^3$ nm.

48. The geometry of $XeOF_4$ by VSEPR theory is :

(1) square pyramidal (2) octahedral (3*) pentagonal planar (4) trigonal bipyramidal

Sol. 

sp^3d^2 hybridized with one position occupied by lone pair.

49. Complete hydrolysis of starch gives :

(1*) glucose only

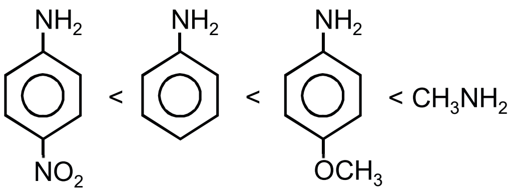
(2) glucose and fructose in equimolar amounts

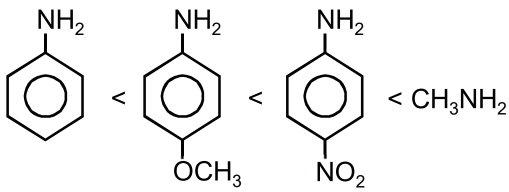
(3) galactose and fructose in equimolar amounts

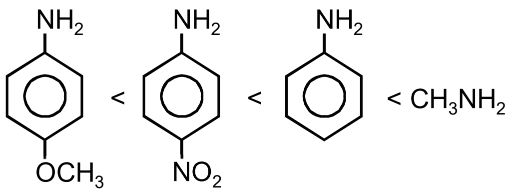
(4) glucose and galactose in equimolar amounts

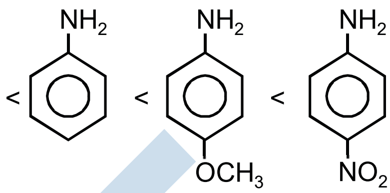
Sol. Starch is a polymer of glucose.

50. Arrange the following amines in the order of increasing basicity.

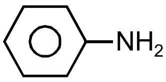
(1*) 

(2) 

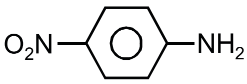
(3) 

(4) CH_3NH_2 < 

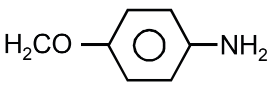
Sol. $CH_3–NH_2$ pKa = 10.64



pKa = 4.62



pKa = 0.98



pKa = 5.29

51. **The reaction :** $2N_2O_5(g) \rightarrow 4NO_2(g) + O_2(g)$ follows first order kinetics. The pressure of a vessel containing only N_2O_5 was found to increase from 50 mm Hg to 87.5 mm Hg in 30 min. The pressure exerted by the gases after 60 min. will be (Assume temperature remains constant) :

(1) 150 mm Hg (2) 116.25 mm Hg (3) 125 mm Hg (4*) 106.25 mm Hg

Sol. $2N_2O_5(g) \rightarrow 4NO_2(g) + O_2(g)$

t = 0 50 0 0

t = 30 50 – 2x 4x x

$\Rightarrow$ 87.5 = 50 + 3x

$\Rightarrow$ 3x = 37.5 $\Rightarrow$ x = 12.5

$\Rightarrow$ $P_{N_2O_5}$ after 30 min = 50 – 25 = 25

$\Rightarrow$ $t_{1/2}$ = 30 min.

Hence after 60 min, (two half lives), $P_{N_2O_5}$ remaining $= \frac{50}{4} = 12.5$ torr.

$\Rightarrow$ Hence decrease in $P_{N_2O_5}$ = 50 – 12.5 = 37.5 torr.

$\Rightarrow$ P_{NO_2} = 2 × 37.5 = 75 torr

$P_{O_2} = \frac{37.5}{2} = 18.75$ torr

$\Rightarrow$ P_{total} = 12.5 + 75 + 18.75

= 106.25 torr.

52. The correct statement on the isomerism associated with the following complex ions,

(a) $[Ni(H_2O)_5NH_3]^{2+}$,(b) $[Ni(H_2O)_4(NH_3)_2]^{2+}$ and (c) $[Ni(H_2O)_3(NH_3)_3]^{2+}$ is

(1) (a) and (b) show geometrical and optical isomerism

(2) (a) and (b) show only geometrical isomerism

(3) (b) and (c) show geometrical and optical isomerism

(4*) (b) and (c) show only geometrical isomerism

Sol. (a) does not show G.I.(b) and (c) show G.I. but all isomers are optically inactive.

53. A solution at 20ºC is composed of 1.5 mol of benzene and 3.5 mol of toluene. If the vapour pressure of pure benzene and pure toluene at this temperature are 74.7 torr and 22.3 torr, respectively, then the total vapour pressure of the solution and the benzene mole fraction in equilibrium with it will be, respectively:

(1) 35.0 torr and 0.480 (2*) 38.0 torr and 0.589

(3) 30.5 torr and 0.389 (4) 35.8 torr and 0.280

Sol. $x_{Benzene} = \frac{1.5}{5} 0.3, x_{totuene} = 0.7$

$\Rightarrow$ P_T = 74.7 × 0.3 × 22.3

= 22.41 + 15.61

= 38.02 torr

and $y_{benzene} = \frac{22.41}{38.02} = 0.589$

54. In the presence of a small amount of phosphorous, aliphatic carboxylic acids react with chlorine or bromine to yield a compound in which α-hydrogen has been replaced by halogen. This reaction is known as :

(1*) Hell-Volhard-Zelinsky reaction (2) Rosenmund reaction

(3) Wolff-Kischner reaction (4) Etard reaction

55. Which of the following is not an assumption of the kinetic theory of gases?

(1) A gas consists of many identical particles which are in continual motion

(2) Collisions of gas particles are perfectly elastic

(3) Gas particles have negligible volume.

(4*) At high pressure, gas particles are difficult to compress.

Sol. No such assumption is made by KTG.

56. The optically inactive compound from the following is :

(1) 2-chloropentane (2) 2-chloropropanal

(3*) 2-chloro-2-methylbutane (4) 2-chlorobutane

Ans. (3)

Sol. $CH_3-CH_2-CH_2-\underset{Cl}{\underset{|}{CH}}-CH_3$

$CH_3-\underset{Cl}{\underset{|}{CH}}-CHO$

$CH_3-CH_2-\underset{Cl}{\underset{|}{\overset{CH_3}{\overset{|}{C}}}}-CH_3$

$CH_3-CH_2-\underset{Cl}{\underset{|}{CH}}-CHO$

57. The following statements relate to the adsorption of gases on a solid surface. Identify the incorrect statement among them :

(1) Enthalpy of adsorption is negative

(2*) On adsorption the residual forces on the surface are increased

(3) Entropy of adsorption is negative

(4) On adsorption decrease in surface energy appears as heat

Sol. Adsorption takes place due to the presence of residual forces on the surface. After adsorption, these are decreased.

58. In the isolation of metals, calcination process usually results in :

(1*) metal oxide (2) metal sulphide (3) metal hydroxide (4) metal carbonate

Sol. Usually calcination results in metal oxides as metal carbonates, hydroxides, all decompose to oxides.

59. If the principal quantum number n = 6, the correct sequence of filling of electrons will be :

(1*) $ns \rightarrow (n-2)f \rightarrow (n-1)d \rightarrow np$

(2) $ns \rightarrow (n-1)d \rightarrow (n-2)f \rightarrow np$

(3) $ns \rightarrow (n-2)f \rightarrow np \rightarrow (n-1)d$

(4) $ns \rightarrow np \rightarrow (n-1)d \rightarrow (n-2)f$

Sol. Following Aufbau principle for filling electrons.

60. In the long form of the periodic table, the valence shell electronic configuration of $5s^25p^4$ corresponds to the element present in :

(1*) Group 16 and period 5

(2) Group 16 and period 6

(2) Group 17 and period 6

(4) Group 17 and period 5

Sol. Valence shell number indicates period number. ns^2np^4 correspond to group 16.

PART-C-MATHEMATICS

61. $\lim\limits_{x\to 0}\dfrac{e^{x^2}-\cos x}{\sin^2 x}$ is equal to

(1*) $\frac{3}{2}$ (2) $\frac{5}{4}$ (3) 2 (4) 3

Sol. $\lim\limits_{x\to\infty}\dfrac{\left(1+x^2+\frac{x^4}{2!}+\frac{x^6}{3!}+.......\right)-\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}+.......\right)}{\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}+......\right)}$

$=\frac{3}{2}$

62. If the tangent to the conic, $y-6=x^2$ at (2, 10) touches the circle, $x^2+y^2+8x-2y=k$ (for some fixed k) at a point (α, β); then (α, β) is

(1) $\left(\frac{-6}{17},\frac{10}{17}\right)$ (2) $\left(\frac{-7}{17},\frac{6}{17}\right)$ (3*) $\left(\frac{-8}{17},\frac{2}{17}\right)$ (4) Q $\left(\frac{-4}{17},\frac{1}{17}\right)$

Sol. y' = 2x at (2, 10), y' = 4
tangent y – 10 = 4(x – 2)
$\Rightarrow y = 4x + 2 \Rightarrow 4x - y + 2 = 0$
Pass $(\alpha, \beta) \Rightarrow 4\alpha - \beta + 2 = 0 \Rightarrow \beta = 4\alpha + 2$ .... (1)
and 2x + 2y y' + 8 - 2 y' = 0

$y'=\dfrac{2x+8}{2-2y}=\dfrac{2\alpha+8}{2-2\beta}=4$ ....(2)

from 1 & 2 we get $\alpha=\frac{-8}{17}, \beta=\frac{2}{17}$

63. A factory is operating in two shifts, day and night, with 70 and 30 workers respectively. If per day mean wage of the day shift workers is Rs. 54 and per day mean wage of all the workers is Rs. 60, then per day mean wage of the night shift workers (in Rs.) is

(1) 75 (2) 69 (3*) 74 (4) 66

Sol. $\dfrac{70x+30y}{100}=60\Rightarrow 3y=600-7x$ $\Rightarrow 3y = 600 - 378$ (x = 54) $\Rightarrow y=\dfrac{222}{3}=74$

64. Let L be the line passing through the point P(1, 2) such that its intercepted segment between the coordinate axes is bisected at P. If L_1 is the line perpendicular to L and passing through the point (–2, 1), then the point of intersection of L and L_1 is

(1*) $\left(\frac{4}{5},\frac{12}{5}\right)$ (2) $\left(\frac{11}{20},\frac{29}{10}\right)$ (3) $\left(\frac{3}{10},\frac{17}{5}\right)$ (4) $\left(\frac{3}{5},\frac{23}{10}\right)$

Sol. Line L is 2x + y = 4
Line L_1 is x – 2y = – 4
intersection point is $\left(\frac{4}{5},\frac{12}{5}\right)$

65. The integral $\int \frac{dx}{(x+1)^{3/4}(x-2)^{5/4}}$ is equal to

(1) $\frac{-4}{3}\left(\frac{x-2}{x+1}\right)^{1/4} + C$

(2) $4\left(\frac{x+1}{x-2}\right)^{1/4} + C$

(3) $4\left(\frac{x-2}{x+1}\right)^{1/4} + C$

(4*) $\frac{-4}{3}\left(\frac{x+1}{x-2}\right)^{1/4} + C$

Sol. $\int \frac{dx}{(x+1)^2\left(\frac{x-2}{x+1}\right)^{5/4}} \Rightarrow \frac{x-2}{x+1} = t \Rightarrow \frac{1}{(x+1)^2}dx = \frac{dt}{3}$

$= \int \frac{dt}{3t^{5/4}} = \frac{-4}{3t^{1/4}} = -\frac{4}{3}\left(\frac{x+1}{x-2}\right)^{\frac{1}{4}} + C$

66. The distance, from the origin, of the normal to the curve, x = 2 cos t + 2t sin t, y = 2 sin t – 2t cos t at $t = \frac{\pi}{4}$, is

(1*) 2 (2) 4 (3) $\sqrt{2}$ (4) $2\sqrt{2}$

Sol. $\frac{dx}{dt} = -2\sin t + 2\sin t + 2t\cos t = 2t\cos t$

$\frac{dy}{dt} = 2\cos t - 2\cos t + 2t\sin t = 2t\sin t$

$\text{so}\frac{dy}{dx} = \tan t$

$\text{slope of normal} = -\frac{1}{\tan t}$

$m|_{t=\frac{\pi}{4}} = -1$

$\text{equation of normal } a + t = \frac{\pi}{4}$

$y - \sqrt{2} + \frac{\pi}{2\sqrt{2}} = -1\left(x - \sqrt{2} - \frac{\pi}{2\sqrt{2}}\right)$

$x + y - 2\sqrt{2} = 0$

distance of normal from

$= \left|\frac{-2\sqrt{3}}{\sqrt{1+1}}\right| = 2$

67. Let X be a set containing 10 elements and P(X) be its power set. If A and B are picked up at random from P(X), with replacement, then the probability that A and B have equal number of elements, is

(1*) $\frac{^{20}C_{10}}{2^{20}}$ (2) $\frac{(2^{10}-1)}{2^{10}}$ (3) $\frac{^{20}C_{10}}{2^{10}}$ (4) $\frac{(2^{10}-1)}{2^{20}}$

Sol. Total number of subsubsets of set X = 2^{10} = 1024 number of subsets with one element = $^{10}C_1$

Number of subsets with two elements = $^{10}C_2$

:

:

Number of subsets with 10 elements = $^{10}C_{10}$

A & B are taken from P(X) from 210 subsets so total ways = $2^{10}.2^{10}$

Number of ways such that A and B have equal number of elements =

$(^{10}C_0)^2 + (^{10}C_1)^2 + (^{10}C_2)^2 + \ldots\ldots + (^{10}C_{10})\ 2$

$= {}^{20}C_{10}$

Probability = $\frac{^{20}C_{10}}{2^{10}.2^{10}}$

68. If Rolle's theorem holds for the function $f(x) = 2x^3 + bx^2 + cx$, $x \in [-1, 1]$, at the point $x = \frac{1}{2}$, then 2b + c equals

(1) 1 (2*) –1 (3) 2 (4) –3

Sol. f(1) = f(–1)

$2 + b + c = -2 + b - c \Rightarrow c = -2$

$f'(x) = 6x^2 + 2bx + c$

$= 6\left(\frac{1}{2}\right)^2 + 2b\left(\frac{1}{2}\right) + C$

$= \frac{3}{2} + b = c = 0 \Rightarrow b = \frac{1}{2}$

Now 2b + c = 1 – 2 = – 1

69. The area (in square units) of the region bounded by the curves $y + 2x^2 = 0$ and $y + 3x^2 = 1$, is equal to

(1) $\frac{3}{5}$ (2*) $\frac{4}{3}$ (3) $\frac{3}{4}$ (4) $\frac{1}{3}$

Sol. $\left.\begin{matrix} y + 2x^2 = 0 \\ \text{and} \quad y + 3x^2 = 1 \end{matrix}\right\}$ are parabola

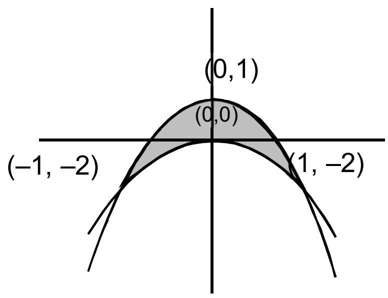
point of intersection of these two curves are (1, -2) & (-1, -2)

Area bounded by these two curves = $2\int_0^1 \left\{\left(1 - 3x^2\right) - \left(-2x^2\right)\right\} dx$

$= 2\int_0^1 \left(1 - x^2\right) dx$

$= 2\left|x - \frac{x^3}{3}\right|_0^1$

$= \left(\frac{2}{3} - 0\right)$



$= \frac{4}{3}$

70. Let the sum of the first three terms of an A.P. be 39 and the sum of its last four terms be 178. If the first term of this A.P. is 10, then the median of the A.P. is

(1) 31 (2*) 29.5 (3) 28 (4) 26.5

Sol. $10 + (10 + d) + (10 + 2d) = 39 \Rightarrow d = 3$

$t_n = 10 + (n - 1)3 = 3n + 7$

Also $(3n + 7) + (3n - 3 + 7) + (3n - 9 + 7) = 178$

$\Rightarrow \quad n = 14$

$\therefore \quad \text{median} = \frac{t_7 + t_8}{2} = \frac{28 + 31}{2} = 29.5$

71. In a certain town, 25% of the families own a phone and 15% own a car; 65% families own neither a phone nor a car and 2,000 families own both a car and a phone. Consider the following three statements:

(a) 5% families own both a car and a phone

(b) 35% families own either a car or a phone

(c) 40,000 families live in the town

Then,

(1) All (a), (b) and (c) are correct (2*) Only (a) and (c) are correct

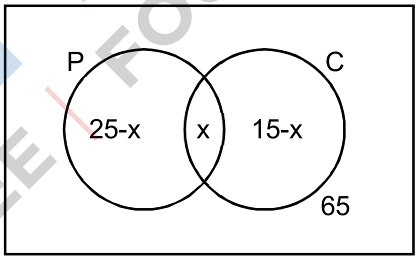
(3) Only (b) and (c) are correct (4) Only (a) and (b) are correct

Sol. $65 + 25 - x + x + 15 - x = 100$

$\Rightarrow 105 - x = 100 \Rightarrow x = 5$

and $\frac{5}{100} \times 40000 = 2000$

family live in the town.



72. The contrapositive of the statement "If it is raining, then I will not come", is:

(1*) If I will come, then it is not raining. (2) If I will not come, then it is not raining.

(3) If I will come, then it is raining. (4) If I will not come, then it is raining.

Sol. Let p : It is raining

q : I will not come

contrapositive of $p \rightarrow q$

is $\sim q \rightarrow \sim p$

$\Rightarrow$ If I will come then if is not raining

73. If $y + 3x = 0$ is the equation of a chord of the circle, $x^2 + y^2 - 30x = 0$, then the equation of the circle with this chord as diameter is:

(1) $x^2 + y^2 + 3x - 9y = 0$ (2) $x^2 + y^2 + 3x + 9y = 0$

(3*) $x^2 + y^2 - 3x + 9y = 0$ (4) $x^2 + y^2 - 3x - 9y = 0$

74. If $f(x) = 2\tan^{-1} x + \sin^{-1}\left(\frac{2x}{1+x^2}\right)$, x > 1 then f(5) is equal to

(1) $\frac{\pi}{2}$ (2) $4\tan^{-1}(5)$ (3) $\tan^{-1}\left(\frac{65}{156}\right)$ (4*) π

Sol. $x^2 + y^2 - 30x + \lambda(y + 3x) = 0$

$\text{centre} \equiv \left[-\frac{3\lambda - 30}{2}, -\frac{\lambda}{2}\right]$

centre lies on y + 3x = 0

$\Rightarrow \lambda = 9$

circles is $x^2 + y^2 - 3x + 9y = 0$

75. If 2 + 3i is one of the roots of the equation $2x^3 - 9x^2 + kx - 13 = 0$, $k \in R$, then the real root of this equation

(1) exists and is equal to 1 (2) does not exist

(3) exists and is equal to $\frac{-1}{2}$ (4*) exists and is equal to $\frac{1}{2}$

$$2 + 3i + 2 - 3i + \alpha = \frac{9}{2}$$

$$\alpha = \frac{9}{2} - 4 = \frac{1}{2}$$

76. The least value of the product xyz for which the determinant $\begin{vmatrix} x & 1 & 1 \\ 1 & y & 1 \\ 1 & 1 & z \end{vmatrix}$ is non negative, is

(1*) -8 (2) –1 (3) $-16\sqrt{2}$ (4) $-2\sqrt{2}$

Sol. $\begin{vmatrix} x & 1 & 1 \\ 1 & y & 1 \\ 1 & 1 & z \end{vmatrix} \geq 0$

$\Rightarrow xyz + 2 - y - x - z \geq 0$

$\Rightarrow xyz + 2 \geq x + y + z \geq 3(xyz)^{1/3}$

put $(xyz)^{1/3} = t$

$\Rightarrow t3 - 3t + 2 \geq 0$

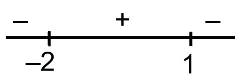
$\Rightarrow (t - 1)(t2 + t - 2) \geq 0$

$\Rightarrow (t - 1)2(t + 2) \geq 0$

$\Rightarrow t \geq -2$

$\Rightarrow (xyz)^{1/3} \geq -2$

$\Rightarrow xyz \geq -8$



77. In a Δ ABC, $\frac{a}{b} = 2 + \sqrt{3}$ and ∠C = 60°. Then the ordered pair (∠A, ∠B) is equal to

(1) (75°, 45°) (2) (45°, 75°) (3) (15°, 105°) (4*) (105°, 15°)

Sol. $\frac{a}{b}=\frac{2+\sqrt{3}}{1}$ $\quad\therefore\quad$ $\frac{a+b}{a-b}=\frac{3+\sqrt{3}}{\sqrt{3}+1}=\sqrt{3}$

$\tan\frac{A-B}{2}=\frac{a-b}{a+b}\cot\frac{C}{2}=\frac{1}{\sqrt{3}}\cot 30^\circ=1$

$\frac{A-B}{2}=45 \quad\Rightarrow\quad A+B=120 \quad A=105^\circ, B=15^\circ$

78. If $A=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$, then which one of the following statements is not correct?

(1*) $A^2+I=A(A^2-I)$ (2) $A^4-I=A^2+1$

(3) $A^3+I=A(A^3-I)$ (4) $A^3-I=A(A-I)$

Sol. $A^2=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}=\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}=-I$

$A^3=-A$

$A^4=-A^2=I$

$A^5=A$

Now

(1) $A^3-I=-A-I$

$A(A-I)=A^2-A=-I-A$

(2) $A^3+I=-A+I$

$A(A^3-I)=A(-A-I)=-A^2-A=I-A$

(3) $A^2+I=-A(I+I)=-2A$

$A^2+I\neq A(A^2-I)$

(4) $A^4-I=I-I=0$

$A^2+I=-I+I=0$

79. For $x>0$, let $f(x)=\int_1^x \frac{\log t}{1+t}dt$. Then $f(x)+f\left(\frac{1}{x}\right)$ is equal to

(1*) $\frac{1}{2}(\ell og\, x)^2$ (2) $\frac{1}{4}(\ell og\, x)^2$ (3) $\frac{1}{4}\ell og\, x^2$ (4) log x

Sol. $f(x)+f\left(\frac{1}{x}\right)=\int_1^x \frac{\log t}{t}dt=\frac{(\log x)^2}{2}$

80. The value of $\sum_{r=16}^{30}(r+2)(r-3)$ is equal to

(1) 7775 (2*) 7780 (3) 7785 (4) 7770

Sol. $\text{Given}=\sum_{r=16}^{30}(r^2-r-6)=\sum_{r=1}^{30}(r^2-r-6)-\sum_{r=1}^{15}(r^2-r-6)$

$=\left(\sum_{r=16}^{30} r^2-\sum_{r=1}^{15} r^2\right)-\left(\sum_{r=1}^{30}\ -\sum_{r=1}^{15} r\right)-6(30-15)$

= 8215 – 345 - 90 = 7780

81. The points $\left(0, \frac{8}{3}\right)$, (1, 3) and (82, 30)

(1) form a right angled triangle

(2) form an obtuse angled triangle

(3) form an acute angled triangle

(4*) lie on a straight line

Sol. $AB = \sqrt{(1-0)^2 + \left(3-\frac{3}{8}\right)^2} = \frac{\sqrt{10}}{3}$

$BC = \sqrt{(1-0)^2 + (30-3)^2} = 27\sqrt{10}$

$CA = \sqrt{(82-0)^2 + \left(30-\frac{8}{3}\right)^2} = \frac{82\sqrt{10}}{3}$

Clearly AB + BC = CA ∴ A,B,C are collinear

82. The largest value of r for which the region represented by the set $\{\omega \in C / |\omega - 4 - i| \le r\}$ is contained in the region represented by the set $\{z \in C / |z - 1| \le |z - i|\}$, is equal to

(1) $\sqrt{17}$

(2) $\frac{3\sqrt{2}}{2}$

(3*) $\frac{5\sqrt{2}}{2}$

(4) $2\sqrt{2}$

Sol. $|w - 4 - i| \le r \Rightarrow$ circle centre (4, 1) radiums = r

$|z - 1 \le |z + i| \Rightarrow$ straight line y = -x

$\therefore \quad \text{maximum } r = \frac{4+1}{\sqrt{1+1}} = \frac{5}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$

83. An ellipse passes through the foci of the hyperbola, $9x^2 - 4y^2 = 36$ and its major and minor axes lie along the transverse and conjugate axes of the hyperbola respectively. If the product of eccentricities of the two conics is $\frac{1}{2}$, then which of the following points does not lie on the ellipse?

(1) $\left(\sqrt{13}, 0\right)$

(2*) $\left(\frac{\sqrt{13}}{2}, \frac{\sqrt{3}}{2}\right)$

(3) $\left(\frac{\sqrt{39}}{2}, \sqrt{3}\right)$

(4) $\left(\sqrt{\frac{13}{2}}, \sqrt{6}\right)$

Sol. $\frac{x^2}{4} - \frac{y^2}{9} = 1$

focii are $(\sqrt{13}, 0)$ and $\left(-\sqrt{13}, 0\right)$

eccentricity of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \Rightarrow e_E^2 = 1 - \frac{b^2}{a^2}$

$\Rightarrow \frac{1}{13} = 1 - \frac{b^2}{a^2} \qquad \Rightarrow \frac{b}{a} = \sqrt{\frac{12}{13}} \qquad .....(1)$

Ellipse passes through $\left(\pm\sqrt{13}, 0\right) \Rightarrow \frac{13}{a^2} = 1 \qquad \Rightarrow a^2 = 13 \qquad \Rightarrow b = \sqrt{12}$

Equation of ellipse $\equiv \frac{x^2}{13} + \frac{y^2}{12} = 1$

which is passes through $(\sqrt{13},0),\left(\sqrt{\frac{13}{2}},\sqrt{6}\right)$ and $\left(\frac{\sqrt{39}}{2},\sqrt{3}\right)$

84. The number of ways of selecting 15 teams from 15 men and 15 women, such that each team consists of a man and a woman, is

(1*) 1240 (2) 1880 (3) 1120 (4) 1960

Sol. $\frac{15\times16\times31}{6}=1240$

85. If y(x) is the solution of the differential equation $(x + 2)\frac{dy}{dx} = x^2 + 4x - 9$, $x \neq -2$ and $y(0) = 0$, then $y(-4)$ is equal to

(1) 1 (2) –1 (3) 2 (4*) 0

Sol. $(x+2)\frac{dy}{dx}=(x+2)^2-13 \quad \frac{dy}{dx}=(x+2)-\frac{13}{x+2}$

$y=\frac{x^2}{2}+2x-13\ell n(x+2)+C$ at $x=0, y=0 \Rightarrow c=13\ell n2$

$y=\frac{x^2}{2}+2x-13\ell n|x+2|+13\ell n2=0$

86. If the coefficients of the three successive terms in the binomial expansion of $(1 + x)^n$ are in the ratio 1 : 7 : 42, then the first of these terms in the expansion is

(1) 6^{th} (2) 9^{th} (3) 8^{th} (4*) 7^{th}

Sol. ${}^nC_{r-1} : {}^nC_r : {}^nC_{r+1}$

$\frac{r}{n-r+1}=\frac{1}{7}$

$\Rightarrow 8r = n + 1$

$\frac{r+1}{n-r}=\frac{1}{6}$

$\Rightarrow 7r = n - 6$

$\Rightarrow r = 7$

87. Let the tangents drawn to the circle, $x^2 + y^2 = 16$ from the point P (0, h) meet the x-axis at points A and B. If the area of Δ APB is minimum, then h is equal to

(1*) $4\sqrt{2}$ (2) $3\sqrt{3}$ (3) $4\sqrt{3}$ (4) $3\sqrt{2}$

88. If the points $(1, 1, \lambda)$ and $(-3, 0, 1)$ are equidistant from the plane, $3x + 4y - 12z + 13 = 0$, then λ satisfies the equation

(1) $3x^2 - 10x + 21 = 0$ (2) $3x^2 + 10x + 7 = 0$

(3) $3x^2 + 10x - 13 = 0$ (4*) $3x^2 - 10x + 7 = 0$

Sol. $|3 + 4 - 12\lambda + 13| = |-9 + 0 - 12 + 13|$

$\Rightarrow |20 - 12\lambda| = 8$

$\Rightarrow 12\lambda - 20 = \pm 8$

$\Rightarrow 12\lambda = 20 \pm 8$

$\Rightarrow \quad \lambda = 1, \frac{7}{3}$

$\lambda = 1 \text{ or } \lambda = \frac{7}{3} \Rightarrow x^2 - \left(1 + \frac{7}{3}\right)x + \frac{7}{3} = 0$

$3x^2 - 10x + 7 = 0$

89. If the shortest distance between the lines $\frac{x-1}{\alpha} = \frac{y+1}{-1} = \frac{z}{1}$, $(\alpha \neq -1)$ and $x + y + z + 1 = 0 = 2x - y + z + 3$ is $\frac{1}{\sqrt{3}}$, then a value of α is

(1) $\frac{-16}{19}$ (2*) $\frac{32}{19}$ (3) $\frac{19}{32}$ (4) $\frac{-19}{16}$

Sol. Any plane $x + y + z\ (2x - y + z + 3) = 0$

$\Rightarrow (2\lambda + 1)x + (1 - \lambda)y + (1 + \lambda)z + 3\lambda + 1 = 0$

parallel to given line if

$\alpha\ (2\lambda + 1) - 1(1 - \lambda) + 1.(1 + \lambda) = 0$

90. Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that $|\vec{a} + \vec{b}| = \sqrt{3}$. If $\vec{c} = \vec{a} + 2\vec{b} + 3(\vec{a} \times \vec{b})$, then 2 is equal to

(1) $\sqrt{37}$ (2) $\sqrt{51}$ (3*) $\sqrt{55}$ (4) $\sqrt{43}$

Sol. $\left|\vec{a} + \vec{b}\right| = \sqrt{3}$

$\Rightarrow \left|\vec{a} + \vec{b}\right|^2 = 3$

$\Rightarrow 1 + 1 + 2.1.1 \cos\theta = 3$

$\Rightarrow \cos\theta = \frac{1}{2}$

$\Rightarrow \theta = \frac{\pi}{3}$ ......(1)

$\because \left|\vec{a} + \vec{b}\right|^2 = \left|\vec{a}\right|^2 - \left|\vec{a}.\vec{b}\right|^2$

$\Rightarrow \left|\vec{a} + \vec{b}\right|^2 = 1 - \left(\frac{1}{2}\right)^2$

$\Rightarrow \left|\vec{a} + \vec{b}\right|^2 = \frac{\sqrt{3}}{2}$

$\left|\vec{c}\right| = \left|\vec{a} + 2\vec{b} + 3\left(\vec{a} \times \vec{b}\right)\right|^2$

$= (\vec{a} + 2\vec{b} + 3\,|\vec{a} \times \vec{b}\,|).(\vec{a} + 2\vec{b} + 3(\vec{a} \times \vec{b}))$

$= |\vec{a}|^2 + 4\vec{a}.\vec{b} + 9\,|\vec{a} \times \vec{b}|^2 + 4\,|\vec{b}|^2$

$= 1 + 4 + 4\left(\frac{1}{2}\right) + 9\left(\frac{3}{4}\right)$

$= 7 + \frac{27}{4} = \frac{55}{4}$

$|\vec{c}| = \frac{\sqrt{55}}{2} \Rightarrow 2\,|\vec{c}| = \sqrt{55}$